Name ________________________________

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Equation of Line - Review

■ You should be able to do all of the following

- 1. Find the slope of a line given two points on the line.
- 2. Find the slope of a line given the equation of the line.
- 3. Graph a line given the equation of the line.
- 4. Graph a line given its slope and a point on the line.
- 5. Use slope to decide whether two lines are parallel or perpendicular.
- 6. Use slope to decide whether three points lie on a line.
- 7. Find the slope and y-intercept of a line given its equation.
- 8. Know and apply the three forms of the linear equation.
- 9. Write the equation of a line given its slope and y-intercept.
- 10. Write the equation of a line given its slope and a point on the line.
- 11. Write the equations of lines that are horizontal or vertical.
- 12. Write the equation of a line given two points on the line.
- 13. Write the equation of a line through a given point and parallel or perpendicular to a given line.
- 14. Determine the point of intersection of two lines given the equation for each of the lines.
- 15. Find the slope, the x-intercept, and the y-intercept of a line given whose equation is given in any of the three forms. This includes the special cases of a vertical or a horizontal line.

- **Forms of the linear equation**

Standard form	$a x + b y = c$
Slope-intercept form	$y = m x + b$
Point-slope form	$y - y_1 = m(x - x_1)$

Typical problems and their solutions

- 1. Find the slope of a line through $P(-1, 3)$, $Q(4, 7)$.

Solution:

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{4+1} = \frac{4}{5}$.

- 2. Find the slope of the line whose equation is $3x + 2y = 7$.

Solution:

$$3x + 2y = 7 \Longleftrightarrow y = \frac{-3}{2} + \frac{7}{2}$$

From which we see the slope is $\frac{-3}{2}$.

- 3a. Graph a line whose equation is $3x + 2y = 7$.

Solution:

Substitute $x = 0$ to obtain point $P(0, \frac{7}{2})$ and substitute $y = 0$ to obtain point $Q(\frac{7}{3}, 0)$. Plot points P and Q, then draw a line through them.

- 3b. Graph a line whose equation is $y = -\frac{3}{2}x + \frac{7}{2}$.

Solution:

Note that the slope is $\frac{-3}{2}$ and that the y-intercept is $\frac{7}{2}$ (hence the point $(0, \frac{7}{2})$ is on the line. Then proceed as in #4 below.

- 4. Graph a line whose slope is $\frac{-3}{2}$ that passes through the point $P(0, \frac{7}{2})$.

Solution:

Plot the point $P(0, \frac{7}{2})$. Use the slope to plot a second point *3* units down from point *P* and 2 units to the right of point *P*.

- 5a. Let line l_1 be given by $y = 3x - 2$ and line l_2 be given by $y = \frac{-1}{3}x + 1$. Is l_1 parallel to l_2?.

Solution:

No, because $m_1 = 3$, $m_2 = \frac{-1}{3}$, $m_1 \neq m_2$.

- 5b. Let line l_1 be given by $y = 3x - 2$ and line l_2 be given by $y = \frac{-1}{3}x + 1$. Is l_1 perpendicular to l_2?.

Solution:

Yes, because $m_1 = 3$, $m_2 = \frac{-1}{3}$, $m_1 m_2 = -1$.

- 6. Do the points $P(-2, -3)$, $Q(2, 3)$, $R(14, 21)$ lie on a single line?.

Solution:

Yes. The line through points P and Q has slope $m_{PQ} = \frac{3+3}{2+2} = \frac{3}{2}$ and the line through points Q and R has slope $m_{QR} = \frac{21-3}{14-2} = \frac{18}{12} = \frac{3}{2}$. Since both lines pass through Q and have the same slope, they are the same line.

- 7. Find the slope and y-intercept of the line $-5x + 3y + 2 = 0$.

Solution:

$-5x + 3y + 2 = 0 \Longleftrightarrow y = \frac{5}{3}x - \frac{2}{3}$. Thus the line's slope is $\frac{5}{3}$ and its y-intercept is $\frac{-2}{3}$.

- 8a. Write in **point-slope form** the equation of a line that includes points $P(0, 3)$, $Q(5, 7)$.

Solution:

Given two points, we typically use the point-slope form of the linear equation to find the equation of the line through the points. Thus,

$y - y_1 = m(x - x_1)$

$y - 3 = m(x - 0)$

$m = \frac{7-3}{5-0} = \frac{4}{5}$

$\therefore\ y - 3 = \frac{4}{5}(x - 0)$

- 8b. Write in **slope-intercept form** the equation of a line that includes points $P(-1, 3)$, $Q(5, 7)$.

Solution:

Given two points, we typically use the point-slope form of the linear equation to find the equation of the line through the points. Thus,

$y - y_1 = m(x - x_1)$

$y - 3 = m(x + 1)$

$m = \frac{7-3}{5+1} = \frac{4}{6} = \frac{2}{3}$

$y - 3 = \frac{2}{3}(x + 1)$ (*The equation in point-slope form.*)

$y = \frac{2}{3}x + \frac{2}{3} + 3$

$\therefore \quad y = \frac{2}{3}x + \frac{11}{3}$ (*The equation in slope-intercept form.*)

- 8c. Write in **standard form** the equation of a line that includes points $P(-1, 3)$, $Q(5, 7)$.

Solution:

Given two points, we typically use the point-slope form of the linear equation to find the equation of the line through the points. Thus,

$y - y_1 = m(x - x_1)$

$y - 3 = m(x + 1)$

$m = \frac{7-3}{5+1} = \frac{4}{6} = \frac{2}{3}$

$y - 3 = \frac{2}{3}(x + 1)$ (*The equation in point-slope form.*)

$3y - 9 = 2(x + 1)$

$\therefore \quad -2x + 3y = 11$ (*The equation in standard form.*)

- 9. Write the equation of a line whose slope is 2 and which crosses the y-axis at $y = -\frac{3}{5}$. Answer in **slope-intercept** form.

Solution:

Given a slope and an intercept, we typically use the slope-intercept form of the linear equation to find the equation of the line. Thus,

$y = m\,x + b$

$\therefore \quad y = 2\,x - \frac{3}{5}$

- 10. Write the equation of a line whose slope is 2 and which passes through $P(-2, \frac{-7}{3})$. Answer in **slope-intercept** form.

Solution:

Given a slope and a point, we typically use the point-slope form of the linear equation to find the equation of the line. Thus,

$y - y_1 = m(x - x_1)$

$y - (-\frac{7}{3}) = 2\,(x - (-2))$

$y + \frac{7}{3} = 2\,(x + 2)$ (*The equation in point-slope form.*)

$y + \frac{7}{3} = 2\,x + 4$

$\therefore \quad y = 2\,x + \frac{5}{3}$ (*The equation in slope-intercept form.*)

- 11a. Write the equation of the line through $P(-4, 2)$ parallel to the y-axis.

Solution:

The x-coordinate of every point on this line is -4. The y-coordinate takes all values. Therefore,

$x = -4$ is the equation of this line.

- 11b. Write the equation of the line through $P(-4, 2)$ parallel to the x-axis.

Solution:

The y-coordinate of every point on this line is 2. The x-coordinate takes all values. Therefore,

$y = 2$ is the equation of this line.

- 12. Write the equation of the through $P(2, 3)$, $Q(5, 11)$. Answer in **slope-intercept** form

Solution:

Given two points, we typically use the point-slope form of the linear equation to find the equation of the line through the points. Thus,

$y - y_1 = m(x - x_1)$

$y - 3 = m(x - 2)$

$$m = \frac{11-3}{5-2} = \frac{8}{3}$$

$y - 3 = \frac{8}{3}(x - 2)$ (*The equation in point-slope form.*)

$\therefore \quad y - 3 = \frac{8}{3}x + \frac{25}{3}$ (*The equation in slope-intercept form.*)

- 13. Write the equation of a line l_2 through point $P(-7, 3)$ and parallel to $l_1: \ 3x + 11y = 4$. Answer in **standard form**.

We are given a point. The slope must match that of l_1, since the lines are parallel. Our initial equation for l_1 will be in point-intercept form. Then we will rewrite the equation in standard form.

$y - y_1 = m(x - x_1)$

$y - 3 = m(x + 7)$

$$3x + 11y = 4 \Longleftrightarrow y = \frac{-3}{11}x + \frac{4}{11}, \text{ so the slope is } -\frac{3}{11}.$$

$y - 3 = \frac{-3}{11}(x + 7)$

$11y - 33 = -3(x + 7)$

$\therefore \quad 3x + 11y = 12$

- 14a. Determine the point of intersection lines l_1 and l_2 whose equations are respectively $y = 2x - 4$ and $y = 10x - 12$.

Solution:

The point of intersection is a point common to both lines. The coordinates of the point of intersection must satisfy both equations. Finding these coordinates amounts to solving the pair of simultaneous equations

$$\begin{pmatrix} -2x + y = -4 \\ -10x + y = -12 \end{pmatrix}$$

The solution of this pair of equations is $(1, -2)$. Therefore, the lines intersect at the point $P(1, -2)$.

- 14b. Determine the point of intersection lines l_1 and l_2 whose equations are respectively $y = 2x - 4$ and $y = 2x - 12$.

Solution:

It is obvious from the equations that the lines are parallel. Thus, they do not intersect. The answer to this question is "The point of intersection does not exist".

- 15a. Find the slope, the x-intercept, and the y-intercept of a line whose equation is $y = 2x - 7$.

Solution:

The slope is obviously 2.

Since the line crosses the x-axis at $y = 0$, the x-intercept is found by substituting 0 for y in the equation. Doing this, we find that $0 = 2x - 7 \Longrightarrow x = \frac{7}{2}$. Hence, the x-intercept is $\frac{7}{2}$.

Since the line crosses the y-axis at $x = 0$, the y-intercept is found by substituting 0 for x in the equation. Doing this, we find that $y = 2(0) - 7 \Longrightarrow y = -7$. Hence, the y-intercept is -7.

- 15b. Find the slope, the x-intercept, and the y-intercept of a line whose equation is $y = -2$.

Solution:

The line is horizontal, thus it has a slope equal to zero..

Since the line is horizontal, it is parallel to the x-axis. Hence, it never crosses the x-axis. Therefore, the x-intercept does not exist.

Since the y-coordinate of the line is a constant, 2, the line crosses the y-axis at $y = 2$. Therefore, the y-intercept is 2.

- 15c. Find the slope, the x-intercept, and the y-intercept of a line whose equation is $x = -7$.

Solution:

The line is vertical, thus its slope is undefined.

Since the line is vertical, it is parallel to the y-axis. Hence, it never crosses the y-axis. Therefore, the y-intercept does not exist.

Since the x-coordinate of the line is a constant, -7, the line crosses the x-axis at $x = -7$. Therefore, the x-intercept is -7.